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Abstract

Parametric mortality models are aimed to describe the age pattern of mortality in terms of a mathematical function of age. We present a new parametric model to describe mortality for the entire age span: the NIDI mortality model. The model describes mortality in adulthood and advanced age by a mixture of two logistic models. The NIDI mortality model includes four interpretable time-varying parameters, reflecting (i) changes in the shape of the mortality age schedule in young and old age, resulting in compression of mortality, and (ii) a shift of the mortality age schedule to older ages.

Fitting the NIDI model to probabilities of death in 1950 and 2009 for Japan, France, the USA and Denmark, showed a better fit than the well-known Heligman-Pollard model. The four time-varying parameters explain 99 per cent of the change in life expectancy at birth between 1950 and 2009. Shifts in the mortality age schedule explain two thirds of this change.

The NIDI model, thus, is a valid instrument for describing the age pattern of mortality, for disentangling the effects of delay and compression of mortality on the increase in life expectancy, and can serve as a basis for the projection of mortality into the future.

Keywords

Heligman-Pollard model; probability of death; mortality age schedule; rectangularisation of survival curve; compression of mortality; age at death distribution.

Introduction

The age pattern of mortality rates and death probabilities has a similar shape in most countries. Mortality is high in infancy, decreases strongly during early childhood, increases sharply in teenage years (this is often called the "accident hump"), increases exponentially during adulthood and its rate of increase levels off in old age (Lexis 1878; Thiele 1871; Siler 1979; Heligman and Pollard 1980; Tabeau 2001; Engelman et al. 2014). Parametric models are aimed to represent this age pattern by a mathematical model including a limited number of parameters. Parametric models are used to smooth age-specific mortality rates, to make comparisons of mortality across countries and to make forecasts based on an analysis of changes in the values of the parameters (Tabeau 2001).

Gompertz (1825) proposed the first 'law of mortality' describing the exponential increase of mortality in adult ages. Even though the Gompertz model provides an accurate fit for most adult ages, the model tends to underestimate mortality at young adult ages and overestimate mortality at the oldest ages (Vaupel et al., 1998; Bongaarts 2005). To address the underestimation of mortality at young adult ages, Makeham (1860) added a constant. This constant is usually referred to as background or non-senescent mortality which does not vary with age, e.g. deaths caused by accidents and certain infections (Gavrilov and Gavrilova 1991; Horiuchi and Wilmoth 1998; Bongaarts 2005, 2006). Background mortality can change with time but is assumed constant above age 25 or so (Bongaarts 2005).

The Gompertz-Makeham model does not describe the decline of mortality during childhood (Canudas-Romo and Engelman, 2009). Siler (1979) and Rogers and Little (1994) used an exponential model to describe this decline, based on the model proposed by Thiele (1871) (Tabeau 2001; Engelman et al. 2014). However, the difference in the fall of mortality between ages 0 and 1 and between successive ages is much stronger than the decline described by an exponential model. Heligman and Pollard (1980) proposed a double exponential function that provides a more accurate description of the decline in mortality during childhood. The Heligman-Pollard specification includes 3 parameters instead of the 2 parameters used by Thiele (1871), Siler (1979) and Rogers and Little (1994). However, the values of these three parameters are highly correlated which makes it difficult to interpret changes in the values of these parameters (Booth and Tickle 2008).

Thiele (1871) proposed a Gaussian function to describe the excess mortality mortality in early adulthood. Alternatively Heligman and Pollard proposed an asymmetric function for this elevated mortality. Both specifications include three parameters. Kostaki (1992) even added a fourth parameter to this part of the Heligman-Pollard model to improve the fit. Rogers and Little (1994) specified a double exponential function including four parameters as well. Thus each of these models requires relatively many parameters to describe mortality in early adulthood. Furthermore, these specifications assume a decline in mortality after early adulthood. This implies that these models do not include background mortality as suggested by Makeham (1860).

In contrast, the Siler model includes background mortality but ignores the “hump” at early adulthood.

From soon after 30 years of age the probability of dying starts to rise by about 10% with each successive year of age (Thatcher 1999). This increase follows quite closely the Gompertz model. The Thiele, Siler, Rogers-Little and Heligman-Pollard models include an exponential term to describe the increase in mortality in adult age. However, by 80 years the rate of increase slows down (Thatcher 1999). Since the Heligman-Pollard model describes the odds ratio of the probability of dying rather than the death probability itself, this model is able to describe levelling off of the increase in mortality at the oldest ages in contrast with the other three models. Alternatively, the overestimation of mortality at the oldest ages can be addressed by using a logistic model rather than the Gompert model (Kannisto 1994; Thatcher 1999; Thatcher, Kannisto and Vaupel 1998). While the Gompertz model has no limit at the oldest ages, in the logistic model the mortality increase levels off (Bongaarts 2005). Beard (1971), Horiuchi and Wilmoth (1998) and Thatcher et al. (1998) examined complex logistic models with additional parameters, but Thatcher (1999) and Thatcher et al. (1998) conclude that a simple, robust logistic model, the Kannisto model, including only two parameters provides an excellent fit to mortality rates at old ages. The Kannisto model, is used to smooth mortality data included in the Human Mortality Database at the oldest ages (Wilmoth et al. 2007).

This paper presents a new parametric model to describe the age pattern of mortality for the entire age span: the NIDI mortality model. The model requires less parameters to describe infant and childhood mortality than the double exponential term proposed by Heligman and Pollard and provides a more accurate fit than the exponential Thiele, Siler and Rogers-Little models. The model includes a term that describes both the teenage “hump” and the level of background mortality. The model describes mortality in adulthood and advanced age by a mixture of two logistic models. The general form of the NIDI mortality model includes ten parameters, but six of these parameters can be assumed to be constant over time. Thus the model can be used to describe changes in age-specific mortality across time by changes in the values of four parameters only. The four parameters reflect changes in the shape of the mortality age schedule at young and old age, resulting in compression of mortality, and a shift of the age schedule.

We estimate the parameters of the NIDI mortality model by fitting the model to probabilities of death in 1950 and 2009 for four low mortality countries: Japan, France, the USA and Denmark. These four low mortality countries have shown different trends in life expectancy during the last sixty years. In 1950 life expectancy at birth of American and Danish women was 10 years higher than life expectancy of Japanese women, whereas in 2009 Japanese women had a 5 years higher life expectancy than American and Danish women. Since the 1980s Japanese women have had the highest life expectancy in the world. In 1950 Denmark was among the European countries with highest life expectancies, but both Danish women and men have experienced relatively little progress since 1950. In 2009 French women had the highest life expectancy in Europe. While

Japanese and French women have high life expectancies, the gender gap in these countries is relatively big (almost 7 years), whereas Denmark has a small gender gap (4 years).

The NIDI mortality model

The age pattern of mortality can be described by age-specific mortality rates and probabilities of death. The mortality rate is the number of deaths at age *x* divided by the number of person-years at risk at age *x*. The death probability is the probability that a person who has reached age *x* will die before reaching age *x+1*. Age-specific death probabilities can be derived from mortality rates. For example, assuming a uniform distribution of exposure in *x*, $q(x) = m(x) / (1 + ½\, m(x))$ where $q(x)$ is the death probability at age *x* and $m(x)$ is the mortality rate. For one-year intervals and for values below 0.2 the values of mortality rates and death probabilities are close, but death probabilities are always smaller than mortality rates. Death probabilities have a value between 0 and 1, mortality rates may exceed 1. However, even at the oldest ages the value of the mortality rates tends to be lower than 1. The shapes of the age pattern of mortality rates and death probabilities are similar. A mathematical expression that describes the change in mortality rates by age usually gives a good fit to the age pattern of death probabilities as well. One advantage of using death probabilities rather than rates is that probabilities are easy to interpret (King and Soneji 2011). Another advantage is that probabilities rather than rates are used for the calculation of life expectancy using a life table. For that reason we estimate a model to describe the age pattern of probabilities of death. Using rates instead of probabilities would have hardly led to different results. Even though the levels of rates and probabilities differ at the oldest ages, both show an exponential increase in adult age and a levelling off in old age.

Similarly to other models describing mortality during the entire age span, the NIDI mortality model consists of additive components representing mortality at subsequent stages of life. Most models include three components to describe death in infancy and childhood, in adulthood and in old age (Tabeau 2001). The NIDI model includes four terms. Two terms are used to model infant deaths and mortality in early adulthood. Two other terms are included to make a distinction between mortality in adulthood and in old age.

Heligman and Pollard (1980) specify a double exponential term to describe the decline in mortality at young ages: A^{x+B^C}. This term provides a more accurate description of infant and childhood mortality than the exponential model proposed by Thiele (1871), but includes three parameters that tend to be highly correlated, which makes it difficult to estimate the model and to interpret changes in these parameters. We propose a simpler model assuming an inverse relationship of mortality with age: $A / (x + B)$. *A* reflects the level of infant mortality and *B* the rate of decrease of mortality in childhood. The lower the value of *B* the stronger is the decrease in mortality. Since the effect of changes in the value of *B* over time turn out to be relatively small, changes in childhood mortality can mainly be attributed to changes in the value of *A*. This makes interpretation of this term straightforward: lower infant and childhood mortality correspond with a lower value of A.

Heligman and Pollard (1980) describe the "accident hump" of mortality starting in teenage years by an exponential function including three parameters: $Dexp[-E(logx - logF)^2$. This term is similar to the lognormal distribution and describes the sharp increase in teenage years and a gradual decrease in mortality in adult ages. The latter decrease implies that the Heligman-Pollard model does not include background mortality as suggested by Makeham (1860) which describes the part of mortality that does not vary with age (Bongaarts, 2005). We propose an alternative specification which describes the sharp increase in mortality at teenage years (the hump) and assumes a constant level in adult age, and thus accounts for background mortality as well. This pattern can be described by a logistic function: $\frac{ab_0e^{b_0(x-m)}}{1+b_0e^{b_0(x-m)}}$ where b_0 determines how strongly mortality increases at teenage years, m determines at which ages mortality increases strongly, and a is the level of background mortality. Empirical results show that the value of b_0 tends to be close to 1. This implies that m is the age at which half the level of background mortality is reached. A decline in a reflects a decline in background mortality.

Adult mortality increases exponentially. This can be described by a Gompertz or a logistic model. Until advanced age the difference between the Gompertz and logistic model is ignorable (Thatcher 1999). At the oldest ages the Gompertz model tends to overestimate mortality rates. We prefer the logistic model as this is capable of describing the levelling off of the increase in mortality at the oldest ages. The simple logistic model uses one parameter b to describe the increase in mortality by age in adult and old age: $\frac{be^{b(x-M)}}{1+be^{b(x-M)}}$ (Horiuchi et al., 2013). A higher value of b implies lower mortality rates in adult age and higher rates in old age. Thus an increase in b results in compression of mortality around the modal age (Tuljapurkar and Edwards 2011). However, there is no *a priori* reason why compression of mortality below the modal age due to a decrease in mortality at young or adult age should imply that there is compression at the oldest ages as well. In order to account for different developments in adult and old age, our model includes separate terms for mortality in adult and old age. Thus our model allows that changes in the value of b for ages above age x_0 differ from those below x_0. There is another reason for including two terms to describe mortality in adult and advanced age. Although the simple logistic performs reasonably, the fit is not perfect (Bongaarts 2005). There is a small systematic overestimation of mortality between ages 60 and 80 and underestimation at the highest ages in a number of countries (Bongaarts 2005; Himes, Preston, and Condran 1994). Because the fit of the Gompertz and logistic models hardly differs between ages 30 and 80 we could have used a Gompertz term to describe adult mortality and a logistic term to describe mortality in old age. However, in order to make it possible to compare the values of b in adult and advanced ages we use a mixture of two logistics to describe mortality in adult and advanced age.

A change in mortality is usually interpreted as a decline in mortality rates. However Bongaarts (2005) notes that instead of interpreting mortality as falling, the schedule of the mortality rate can be viewed as shifting to higher ages over time. Thus Bongaarts (2005) describes changes in mortality rates by the so-called shifting logistic model. This model implies that mortality rates decline due to a shift in the age schedule rather than due to a change in the value of *b* which changes the shape of the age schedule. Several authors have observed a shifting mortality pattern (Kannisto 1996; Bongaarts and Feeney 2002, 2003; Bongaarts 2005; Cheung, et al. 2005; Cheung and Robine 2007; Canudas- Romo 2008; Thatcher, et al. 2010; Vaupel 2010). For this reason the NIDI mortality model includes a parameter that can describe a shift of the mortality age schedule resulting in delay of mortality to older ages. Horiuchi et al. (2013) show that including the model age at death *M* in the logistic model takes account of shifts of the mortality age schedule: if the age schedule of mortality rates shifts to older ages the modal age at death increases exactly at the same pace as the shift in the mortality curve. Thus we include the modal age at death in the logistic terms of our model that describe adult and old-age mortality. In addition to the modal age at death *M* and the slope parameter *b* the logistic model in advanced ages includes one parameter *g* representing the upper bound of the mortality rates (Horiuchi et al, 2013). The reason for including this parameter is that Gampe (2010) shows that the probability of death is flat beyond age 110 at a value lower than 1.

Thus the NIDI mortality model is defined by:

$$q(x) = \frac{A}{x+B} + \frac{ab_0 e^{b_0(x-m)}}{1+b_0 e^{b_0(x-m)}} + I(x \le x_0)\left[\frac{b_1 e^{b_1(x-M)}}{1+b_1 e^{b_1(x-M)}}\right] + I(x > x_0)\left[\frac{b_2 e^{b_2(x-M)}}{1+\frac{b_2}{g} e^{b_2(x-M)}} + c\right] \qquad (1)$$

where M is the modal age at death, b_1 and b_2 determine the increase in adult mortality below and above age x_0 respectively, $I(.)$ is an indicator function, e.g. $I(x \le x_0) = 1$ if $x \le x_0$ and = 0 if $x > x_0$, g is the upper bound of $q(x)$ and c is a constant that is introduced to avoid 'jumps' in the fitted values at $x = x_0$. We require that the values of the third and fourth terms on the right hand side of (1) are equal at age $x = x_0$. This implies that we assume that

$$\frac{b_1 e^{b_1(x_0-M)}}{1+b_1 e^{b_1(x_0-M)}} = \frac{b_2 e^{b_2(x_0-M)}}{1+\frac{b_2}{g} e^{b_2(x_0-M)}} + c \qquad (2)$$

Thus the value of c can be calculated from the values of b_1, b_2, x_0 and M.

In short, the NIDI mortality model differs in four aspects from previous models. First, we describe infant and childhood mortality by a simpler model than Heligman-Pollard, whereas it provides a better fit than the exponential Thiele specification. Second, we use a logistic model to describe both the excess mortality in young adulthood and the level of background mortality in adult ages. Third, we specify a mixture logistic model with different parameter values in adult and advanced age. Fourth, following Horiuchi et al. (2013) we include the modal age at death as a parameter in the model to describe a shift of the mortality schedule, as suggested by Bongaarts (2005).

Fit of the general form of the NIDI mortality model

We obtained data on annual probabilities of death *q(x)* from the Human Mortality Database (HMDB, 2014). We calculated life table estimates of the age-at-death distribution *d(x)* and the survival rate *l(x)* from the estimates of *q(x)* using the method described by Wilmoth et al. (2007). The estimates of *q(x)* for the oldest ages in the HMDB are smoothed by fitting the Kannisto logistic model to mortality in old age. Since the NIDI mortality model includes a logistic model to describe mortality in advanced age, using smoothed rather than raw data does not affect our estimates. We estimate the model for 1950 and 2009 for Japan, France, the USA and Denmark both women and men.

Usually the parameters of models describing the age pattern of mortality are estimated by fitting the model to age-specific mortality rates or death probabilities rather than to the resulting life table age-at-death distribution. Since we want to examine to what extent changes in death probabilities *q(x)* in young, adult and advanced age lead to a shift in the age-at-death distribution on the one hand and to a change of the shape on the other, we need to assess to what extent our model produces a good fit of the life table age-at-death distribution *d(x)* as well.

Heligman and Pollard estimated the values of the parameters of their model by minimizing the sum of squared errors in the ratios of the estimated and observed values of *q(x)*. This yields similar results as minimizing the squared errors of log*(q(x))*. This method gives relatively high weight to errors at young ages (where the denominator of the ratios is small) and small weight to errors at advanced ages (where the denominator is large). Alternatively one can estimate the parameters by minimizing the squared errors of *q(x)*. This has the opposite effect as fitting the model to log*(q(x))* as it gives small weight to errors at young ages where *q(x)* is low and large weight to errors at high ages where *q(x)* is high. Another alternative is to minimize the squared errors of the life table distribution of age at death *d(x)*. This gives relatively high weight to errors around the modal age at death where *d(x)* is high and small weight to errors at both young and old ages, where *d(x)* is small. Since our model is aimed to describe the entire age pattern of mortality as accurately as possible we define a weighted loss function including errors in *q(x),* log*(q(x))* and *d(x)*. Our loss function is a weighted average of root mean squared errors (rmse) in *q(x)* and log*q(x)* as well as *d(x)*. Since the size of the rmse of *d(x)* tends to be about ten times as low as that of *q(x)* which in turn is about ten times as low as that of log*(q(x))*, we multiply the rmse of *d(x)* by 100 and that of *q(x)* by 10 in order to make their orders of magnitude comparable and to avoid that we would give the errors in log*(q(x))* higher weight than those in *d(x)* and *q(x)*. Second, since we want the model to fit both *d(x)* and *q(x)* we give *d(x)* weight 50 and *q(x)* and log(*q(x)*) each weight 25. Thus our loss function is: 50 *100* rmse[*d(x)*] + 25*rmse[log*(q(x))*] + 25*10*rmse[*q(x)*]. Comparisons of estimates of the model using other weights show that the modal age at death tends to be estimated less accurately if the loss function includes a lower weight of *d(x)* (results not presented here).

Figure 1 illustrates the fit of both the NIDI model and the Heligman-Pollard model to the age-specific probabilities of death for French women in 1950 and 2009. The reason for comparing the fit of the NIDI model with that of the Heligman-Pollard model is that the latter model produces a better fit to the whole age pattern of mortality than the other parametric models discussed above (results not shown here). The upper panel of the figure showing *q(x)* demonstrates that the Heligman-Pollard model tends to overestimate death probabilities in old age, whereas the NIDI mortality model provides a very accurate fit. The lower panel showing the fit of log(*q(x)*) demonstrates that both models provide an accurate fit in young age, but that there is autocorrelation in the errors of the Heligman-Pollard model in adult age for 2009: the model overestimates mortality of women in their thirties, underestimates the increase of mortality of women in their fifties and overestimates the increase of mortality for women in their seventies. Figure 2 shows that as a result the Heligman-Pollard model overestimates the density of deaths between ages 70 and 80 and underestimates the density around the modal age at death. In contrast, the NIDI model provides a very accurate fit of the age-at-death distribution.

<figure 1 about here>

<figure 2 about here>

Fit of the NIDI mortality model with four time-varying parameters

When fitting model (1) to data for Japanese, French, US and Danish women and men in 1950 and 2009 it turns out that the value of b_0 is close to 1 and the value of m is close to 16 years for all countries and for both sexes. Thus the model can be fitted by estimating the values of 8 parameters: A, B, a, M, b_1, b_2, g and x_0. Table 1 shows that the fit of the NIDI mortality model including 8 parameters where the values of b_0 and m are fixed is almost as accurate as that of model (1) including 10 parameters. The model including 8 parameters provides a more accurate fit than the Heligman-Pollard model for mortality in each of the four countries for both sexes in both 1950 and 2009. In addition to fixing the values of b_0 and m for each country and sex the values of B, b_1, g and x_0 do not turn out to vary much between 1950 and 2009. The assumption that b_1 is constant through time corresponds with Bongaarts' finding that the slope parameter of the logistic differs across countries and sexes, but is nearly constant over time for a country (Bongaarts 2005). Thus the probability of death at age x in year t can be described by a model including four time-varying parameters only:

$$q(x,t) = \frac{A_t}{x+B} + \frac{a_t e^{(x-16)}}{1+e^{(x-16)}} + I(x \le x_0)\left[\frac{b_1 e^{b_1(x-M_t)}}{1+b_1 e^{b_1(x-M_t)}}\right] + I(x > x_0)\left[\frac{b_{2t} e^{b_{2t}(x-M_t)}}{1+(b_{2t}/g)e^{b_{2t}(x-M_t)}} + c_t\right] \quad (3)$$

where the parameters A_t, a_t, M_t and b_{2t} vary with time and the parameters B, b_1, g and x_0 are assumed equal in 1950 and 2009. Table 1 shows that except for Japanese men in 1950 the fit of the NIDI model with only four time-varying parameters is better than the fit of the Heligman-Pollard model. Focusing on adult mortality Bongaarts and Feeney (2002) show that the shifting logistic model describes changes over time with only one time-varying parameter. However, Bongaarts (2005) admits that there is some loss in the goodness of fit. We avoid this inaccuracy by allowing b_2 to change over time.

<table 1 about here>

Explaining changes in mortality

Changes in the estimated values of the four time-varying parameters of the NIDI mortality model (equation 3) explain changes in the age pattern of mortality. Table 2 shows the estimated values of the parameters of the NIDI mortality model for both 1950 and 2009. The changes in the values of the time-varying parameters have a clear interpretation. The decline of A represents the effect of the decline in infant mortality by 80 to 90 per cent between 1950 and 2009 in France, the USA and Denmark and even by 97 per cent in Japan. The decline in a reflects the decline in young adulthood and background mortality. He table shows that background mortality has declined less strongly among men than among women. In Japan the levels of both infant and background mortality in 1950 were much higher than in the other three countries but by 2009 the differences have become small. Both the decline in A and in a result in a more rectangular shape of the survival curve below the median age at death. The increase in the value of M implies that the survival curve has shifted to the right. In Japan the average annual increase in the modal age at death has been 0.22 years between 1950 and 2009. Thus mortality in Japan has been delayed by 2.6 months per year. In France the annual increase has equaled 0.18 years, in the USA 0.16 years and in Denmark 0.10 years. The increase in the value of b_2 implies that the rise of mortality rates by age in old age has been higher in 2009 than in 1950. This implies compression of mortality in old age and thus a more rectangular shape of the survival curve at advanced age.

Figure 3 shows that the changes in the four time-varying parameters of the NIDI mortality model are able to describe changes in the age-at-death distribution between 1950 and 2009 in Japan, France, the USA and Denmark for both women and men. Both the shift of the distribution to the right and the change in the shape of the distribution can be explained by the changes in these four variables. Figure 3 shows that particularly for Japanese and French women the modal age at death has risen strongly, while the age-at-death distribution has become much more compressed. In Denmark there has hardly been compression.

< table 2 about here >

< figure 3 about here >

Figure 4 shows how the changes in the values of the four time-varying parameters explain changes in the survival curve between 1950 and 2009. The effects of the values of the four parameters are shown consecutively. The dotted line showing the effect of A demonstrates how the survival curve would have changed between 1950 and 2009 if only the value of A would have changed. This is the effect of the decline in infant mortality. The figure shows that as a result of the decline in infant mortality the age at which the life table survival rate $l(x)$ equals 0.9 has increased from 14 to 41 years among Japanese women, an increase by 27 years between 1950 and 2009. The dashed line showing the effect of a demonstrates the additional effect of the change in a added to the effect of the change in A. The decrease in the teenage excess mortality and in the level of background mortality has led to a further increase in the age at which $l(x)$ = 0.9 from 41 to 56 years among Japanese women, an increase by 15 years. As a result of both these changes the survival curve has become more rectangular, since the survival rates in young ages have increased more strongly than around the median age. The line showing the effect of M shows the shift of the survival curve to the right in addition to the effects of changes in A and a. As a result of this shift the median age at death has increased by 15 years in addition to the increase by 5 years that can be attributed to the decline in A and a. Finally the line showing the effect of b_2 shows that the survival in old age has increased less strongly than what would have been the result of the increase in M. Whereas the increase in M would have led to an increase in the age at which $l(x)$ = 0.10 to x = 102 years if there would not have been compression in advanced age, the actual increase has been to x = 99 years. Thus the survival curve has become more rectangular in old age. Figure 4 shows that the rectangularization of the survival curve in young age has been much stronger in Japan than in the other three countries. Particularly the effect of the decline in background mortality (represented by the decrease of a) has been small in France, the USA and France. Among Danish women the shift in the survival curve has been smaller than in the other three countries. Moreover, the shape of the survival curve has hardly changed.

< figure 4 about here >

From the effects of the changes in the parameter values on the life table survival rates, we can calculate the effects on the changes in life expectancy at birth. Table 3 shows the contribution of the changes in the values of the four time-varying parameters to changes in life expectancy at birth between 1950 and 2009. The table shows that in Japan half of the increase in life expectancy at birth can be attributed to the shift of the mortality schedule to the right, *i.e.* delay of mortality to older ages. In Japan the contribution of the decline in infant and background mortality has been more important than in the other three countries. This decrease occurred mainly between 1950 and 1970 (results not shown here). Since 1970 the relative contribution of the shift of the mortality schedule has been much bigger in Japan. Note that the levels of infant and background mortality in Japan in 1950 were considerably higher than in the other three countries. In France and Denmark two thirds of the increase in life expectancy at birth can be attributed to the shift of the mortality curve, and in the USA even more than 80 per cent. Taken together changes in the four time-varying parameters of the NIDI mortality model explain 99 per cent of the change in life expectancy at birth between 1950 and 2009.

< table 3 about here>

Conclusion and discussion

The empirical results presented in this article show that the NIDI mortality model provides an accurate description of the entire age pattern of mortality in four low mortality countries: Japan, France, the USA and Denmark. The model adequately describes changes in mortality in these countries between 1950 and 2009. The general version of the NIDI model (equation 1) includes 10 parameters, but changes in the age pattern of mortality between 1950 and 2009 in the four countries can be described by changes in the values of four time-varying parameters only. The decision which parameters are assumed to be fixed and which are time-varying is an empirical one and may vary from case to case. Thus the specification of equation (3) may differ in other applications of the model.

Changes in the modal age at death (M) reflect shifts of the mortality schedule and thus describe the effect of delay of mortality to older ages. Changes in the values of the other three time-varying parameters describe changes in the shape of the mortality age schedule. Changes in A describe the effect of the decline in infant mortality and changes in a the effect of the decline in the teenage excess mortality and the level of background mortality. The changes in these two parameters describe the rectangularisation of the survival curve below the median age at death. The value of b_2 describes the increase in mortality in advanced age and changes therein reflect the effect of compression of mortality in old age. While in 1950 the estimated value of b_2, the slope in advanced age, did not differ very much from b_1, the slope in adult age, the value of b_2 has increased considerably since. The higher value of b_2 compared with the value of b_1 corresponds with Thatcher's finding that the increase of mortality slows down at old age. The estimated value of b_1 is close to 0.10 in most countries. This is consistent with Thatcher's finding that from soon after 30 years of age the probability of dying starts to rise by about 10% with each successive year of age (Thatcher 1999). Since the value of b_1 does not differ between 1950 and 2009, the decrease in mortality in middle age can be attributed to the shift in the mortality age schedule rather than to compression of mortality. This confirms the study by Bongaarts (2005).

The NIDI mortality model has several advantages compared with other parametric mortality models. First, the NIDI model uses less parameters to describe infant and childhood mortality than the Heligman-Pollard model. Changes through time in infant and childhood mortality can be contributed to changes in the value of one parameter, whereas the Heligman-Pollard model includes three parameters of which the values are highly correlated. Compared with the exponential model suggested by Siler (1979) and others, the NIDI model provides a better fit. Second, the NIDI model uses less parameters to describe the excess mortality in early adulthood than the Heligman-Pollard and Rogers-Little models. One parameter is sufficient to explain changes through time. Third, since the elevated mortality is described by a logistic model, which implies a constant level beyond early adulthood, the NIDI model includes background mortality, without requiring an additional parameter. Fourth, the NIDI model includes one parameter to describe shifts in adult and old age mortality. As this parameter equals the modal age at death, it is straightforward to interpret changes in its value. Fifth, the NIDI model distinguishes two separate parameters to account for the increase in mortality by age in adult and advanced age. This provides a better fit of mortality in old age. Whereas the increase of mortality by age in adult ages has not changed over time, the increase in advanced ages has. Changes in the value of the parameter describing the increase in mortality by age in advanced age accounts for compression of mortality in advanced age. Sixth, the NIDI model provides an accurate fit of both the age schedule of death probabilities and the age-at-death distribution. Seventh, changes in mortality over time can be explained by changes in the values of four parameters only. Changes in these parameters explain 99 per cent of changes in life expectancy at birth. This makes the model useful for projections.

The empirical results show that more than three quarters of the increase in life expectancy at birth between 1950 and 2009 in France, the USA and Denmark can be attributed to the shift in the mortality age schedule. In Japan the contribution of the shift has been smaller, but that can be attributed to changes in the 1950s and 1960s, where Japan moved from relatively high to low levels of mortality in young age. Since 1970 the shift of mortality has been the main cause of the increase in life expectancy at birth in Japan as well. One benefit of our model is that we estimate delay and compression simultaneously. Thus our estimate of compression is not affected by delay of mortality and vice versa. This is a necessary condition for decomposing changes in life expectancy into the effects of delay and compression. Our results show that postponement of mortality has had a much bigger effect than compression, but compression has had an effect as well. This is in line with the recent results by Rossi et al. (2013). Canudas-Romo (2008) referred to this development as the ‘shifting mortality scenario’.

Our findings have implications for forecasting future changes in mortality. Demographers agree that in low mortality countries the levels of infant and background mortality are so low that no big further declines may be expected (Bongaarts 2006). This implies that future increases in life expectancy will depend on further shifts in the mortality age schedules. Between 1950 and 2009 the age schedules have shifted by some 2 months per year. Since mortality rates in old age have declined less strongly than mortality rates around the median age, life expectancy at birth has increased slightly less strongly than the shift of the survival curve, but the effect of compression of mortality in old age has been relatively small between 1950 and 2009. If the delay of mortality will continue and if the effect of compression in old age will not become much stronger in the future, a substantial further increase in life expectancy can be projected. This would result in a stronger increase in life expectancy than projections produced by the widely applied Lee-Carter model (Lee and Carter, 1992). The latter model projects a decline in mortality rates rather than a shift. As mortality rates in advanced age have declined less strongly than around the modal age at death, the Lee-Carter model projects only small future decline in mortality in old age and thus a strong compression of mortality. Bongaarts (2005) notes that the Lee-Carter projection expects little improvement in mortality at the highest ages and large improvements in the 60-80 age group and argues that the shift model gives more robust long-range projections for the age pattern of mortality than does the Lee-Carter method. The Lee-Carter method extrapolates past trends in mortality rates for each age group at its own exponential rate (Lee and Miller 2001; McNown 1992). In contrast, the shift method ensures that the age structure of mortality remains plausible.

Since 1950 the modal age at death has increased by on average 0.24 years per year for Japanese women. In 2009 the modal age at death for Japanese women was 92.6 years. If the average annual increase between 1950 and 2009 would continue in the future, the modal age at death of Japanese women would reach the level of 100 years in the year 2040. Without further compression of mortality that would result in an increase of life expectancy at birth from 86 years in 2009 to 94 years in 2040. A life expectancy at birth of 100 years could be reached when the modal age at death equals 106 years. Assuming a linear increase in the modal age at death and no compression of mortality in advanced age life expectancy could reach a level of 100 years in the year 2065. However, one may question the assumption that there will be no further compression at advanced age (Fries, 1980). Even though the effect of compression in old age on the increase in life expectancy at birth has been small between 1950 and 2009, it would be possible that the effect of compression may become stronger when the model age increases to an age around 100 years. In that case it would take a longer time for life expectancy at birth of Japanese women to reach the level of 100 years. Note that these 'projections' are made for illustrative purposes only. A 'proper' projection should preferably not be based on comparing parameter values in 1950 and 2009 only. Rather one should look at the time series of the estimated parameter values between 1950 and 2009 and use some time series model to project the parameter values of the NIDI mortality model into the future. Using a stochastic model, such as an ARIMA model, one can assess the degree of uncertainty of the forecasts by calculating forecast intervals in a similar way as Lee and Carter (1992) proposed. The NIDI model, thus, can serve as a basis for the projection of mortality into the future.

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Table 1. Loss* of three specifications of the NIDI mortality model compared to the Heligman-Pollard model, four low-mortality countries, 1950 and 2009, by sex

		Heligman-Pollard model	NIDI mortality model		
		8 parameters	10 parameters (equation 1)	8 parameters** (equation 1)	4 time-varying parameters *** (equation 3)
		Women			
Japan	1950	5.13	3.59	4.18	4.54
	2009	9.23	3.24	5.11	7.41
France	1950	6.60	2.20	2.36	3.23
	2009	11.88	4.16	4.28	4.68
USA	1950	7.25	3.48	3.59	3.59
	2009	7.56	1.96	1.98	3.99
Denmark	1950	13.13	5.16	5.26	6.11
	2009	10.88	6.56	6.54	7.04
		Men			
Japan	1950	5.05	4.01	4.82	5.95
	2009	6.58	2.64	2.72	3.24
France	1950	5.20	3.04	3.10	3.50
	2009	12.81	5.07	5.15	5.86
USA	1950	5.33	3.78	3.84	4.22
	2009	9.95	3.15	3.27	3.76
Denmark	1950	12.34	4.74	4.72	5.19
	2009	10.53	5.89	5.89	6.63

*Loss = 50 *100* rmse[$d(x)$] + 25*rmse[log($q(x)$)] + 25*10*rmse[$q(x)$]

**Values of b_0 and m are fixed: b_0 = 1 and m = 16.

*** Values of B, b_1, x_0 and g are assumed equal in 1950 and 2009

Table 2. Estimated values of the parameters of the NIDI mortality model (equation 3), four low-mortality countries, 1950 and 2009, by sex

	Japan		France		USA		Denmark	
	women							
	1950	2009	1950	2009	1950	2009	1950	2009
A	0.0180	0.0005	0.0038	0.0003	0.0028	0.0005	0.0023	0.0003
B	0.3438	0.3438	0.0847	0.0847	0.1018	0.1018	0.0917	0.0917
a	0.0034	0.0002	0.0008	0.0001	0.0006	0.0003	0.0003	0.0001
b_0	1	1	1	1	1	1	1	1
m	16	16	16	16	16	16	16	16
M	78.3	92.6	80.9	90.7	80.1	89.0	80.1	87.3
b_1	0.1069	0.1069	0.0946	0.0946	0.0953	0.0953	0.1004	0.1004
b_2	0.1140	0.1544	0.1216	0.1598	0.1021	0.1331	0.1238	0.1325
x_0	76.1	76.1	56.8	56.8	76.0	76.0	64.0	64.0
g	0.6511	0.6511	0.6294	0.6294	0.6372	0.6372	0.6578	0.6578
	men							
A	0.0172	0.0005	0.0040	0.0003	0.0026	0.0005	0.0031	0.0001
B	0.2911	0.2911	0.0688	0.0688	0.0721	0.0721	0.0873	0.0873
a	0.0033	0.0004	0.0007	0.0005	0.0007	0.0006	0.0005	0.0003
b_0	1	1	1	1	1	1	1	1
m	16	16	16	16	16	16	16	16
M	74.3	86.2	75.4	86.6	75.4	84.8	79.0	83.8
b_1	0.1013	0.1013	0.0873	0.0873	0.0846	0.0846	0.0967	0.0967
b_2	0.1058	0.1195	0.1079	0.1352	0.0900	0.1179	0.1208	0.1240
x_0	74.0	74.0	74.0	74.0	78.0	78.0	69.0	69.0
g	0.6572	0.6572	0.6783	0.6783	0.6951	0.6951	0.6678	0.6678

Note. The values of B, b_1, x_0 and g are assumed equal in 1950 and 2009

Table 3. Decomposition of changes in life expectancy at birth from 1950 to 2009 (in years), based on the NIDI mortality model (equation 3), four low-mortality countries, by sex.

	Japan		France		USA		Denmark	
	women	men	women	men	women	men	women	men
total observed change	25.5	22.0	15.3	14.4	10.0	10.7	9.5	7.7
due to								
delay (change in M)	13.8	11.5	10.1	10.2	8.6	8.8	7.2	4.5
childhood (change in A)	7.1	7.1	3.9	4.6	2.2	2.5	2.2	3.2
young adulthood (change in a)	5.5	4.1	1.2	0.4	0.3	0.1	0.4	0.4
old age (change in b_2)	-0.9	-0.4	0.1	-0.6	-0.9	-0.7	-0.2	-0.1
unexplained	-0.1	-0.2	0.0	-0.2	-0.2	0.0	-0.1	-0.2

Figure 1. Observed and fitted probability of death, comparison between the NIDI mortality model (equation 1) and the Heligman-Pollard model, French women, 1950 and 2009

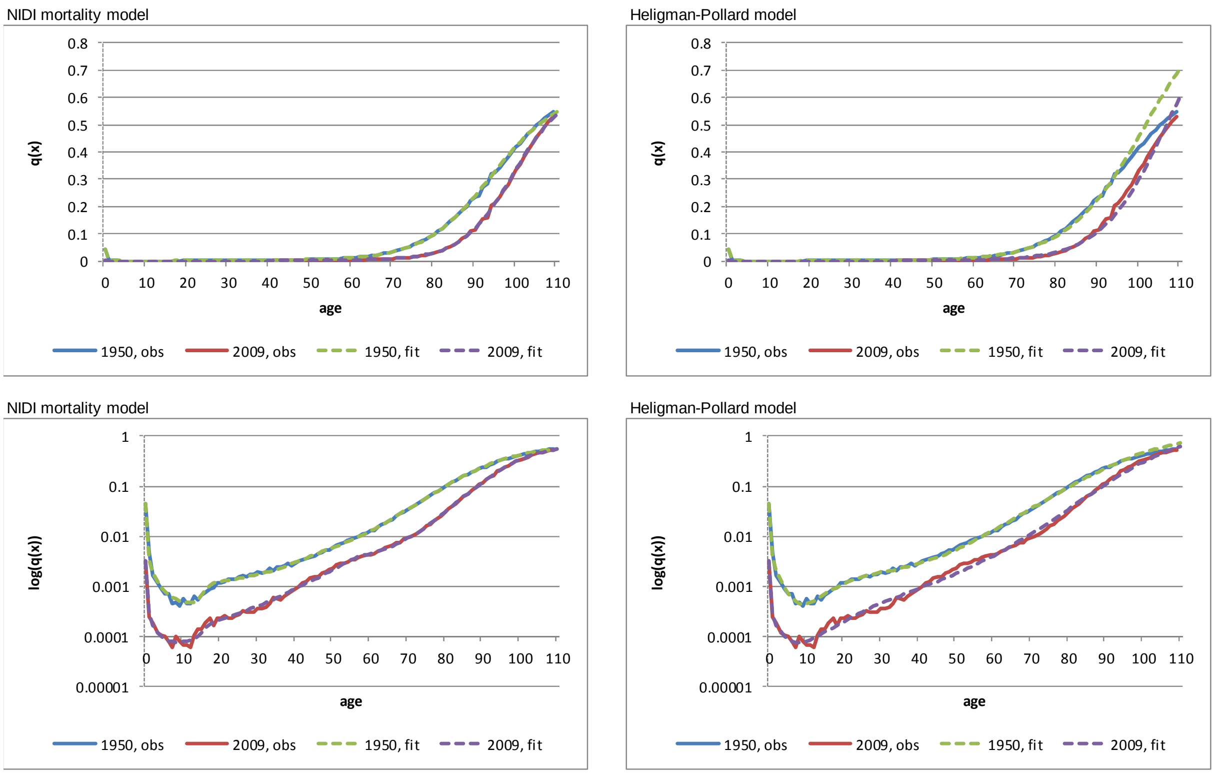


Figure 2. observed and fitted distribution of age at death, comparison between the NIDI mortality model (equation 1) and the Heligman-Pollard model, French women, 1950 and 2009, observed and fitted

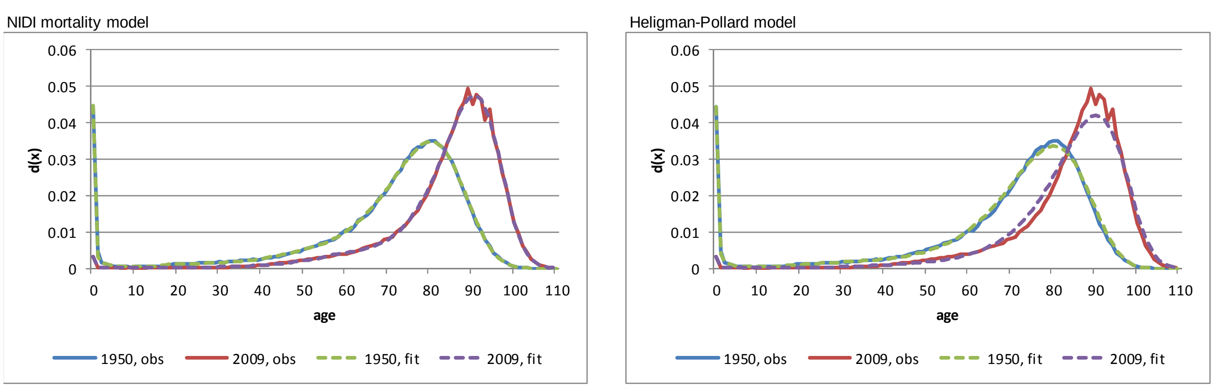
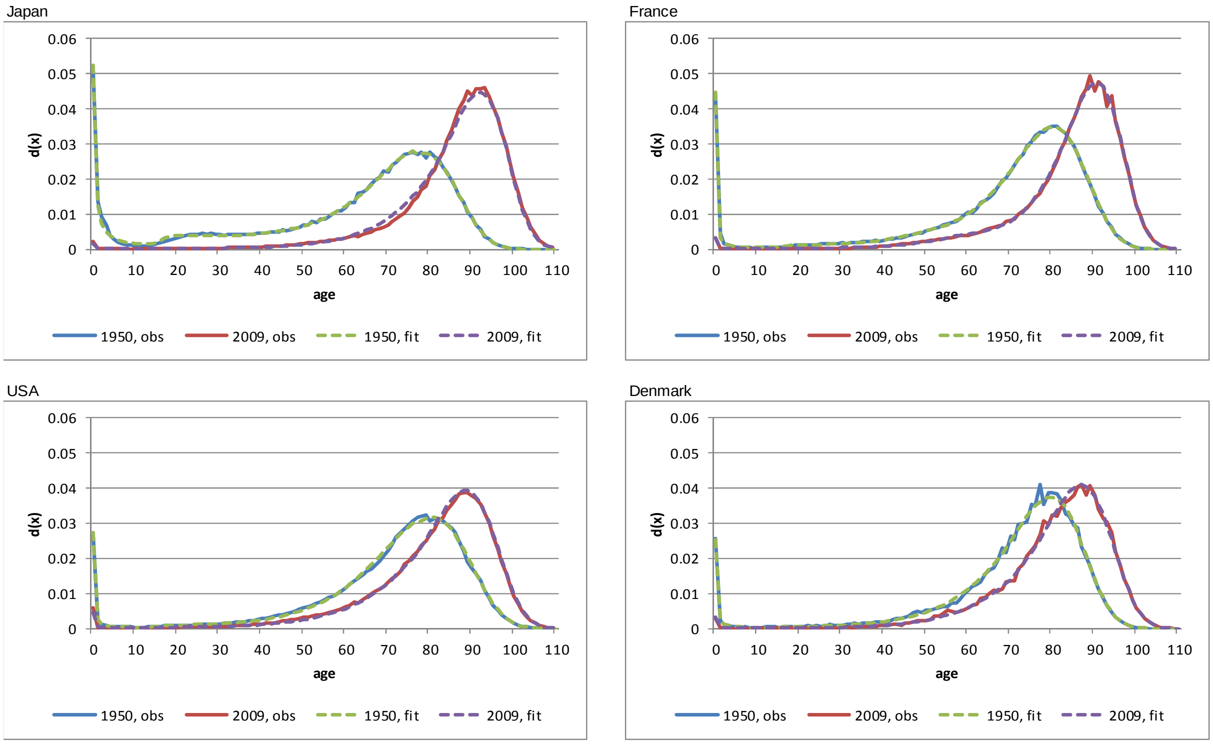


Figure 3. Observed and fitted distribution of age at death, NIDI mortality model (equation 3), four low-mortality countries, 1950 and 2009, by sex

a. Women



b. Men

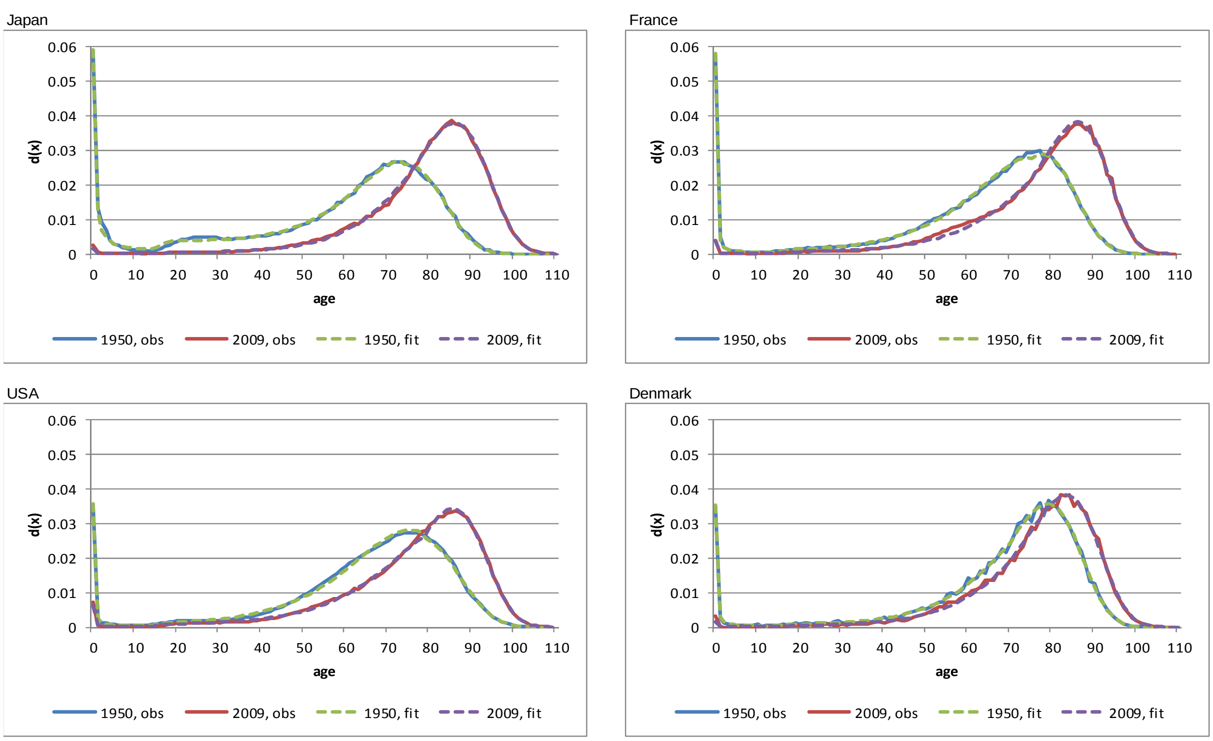
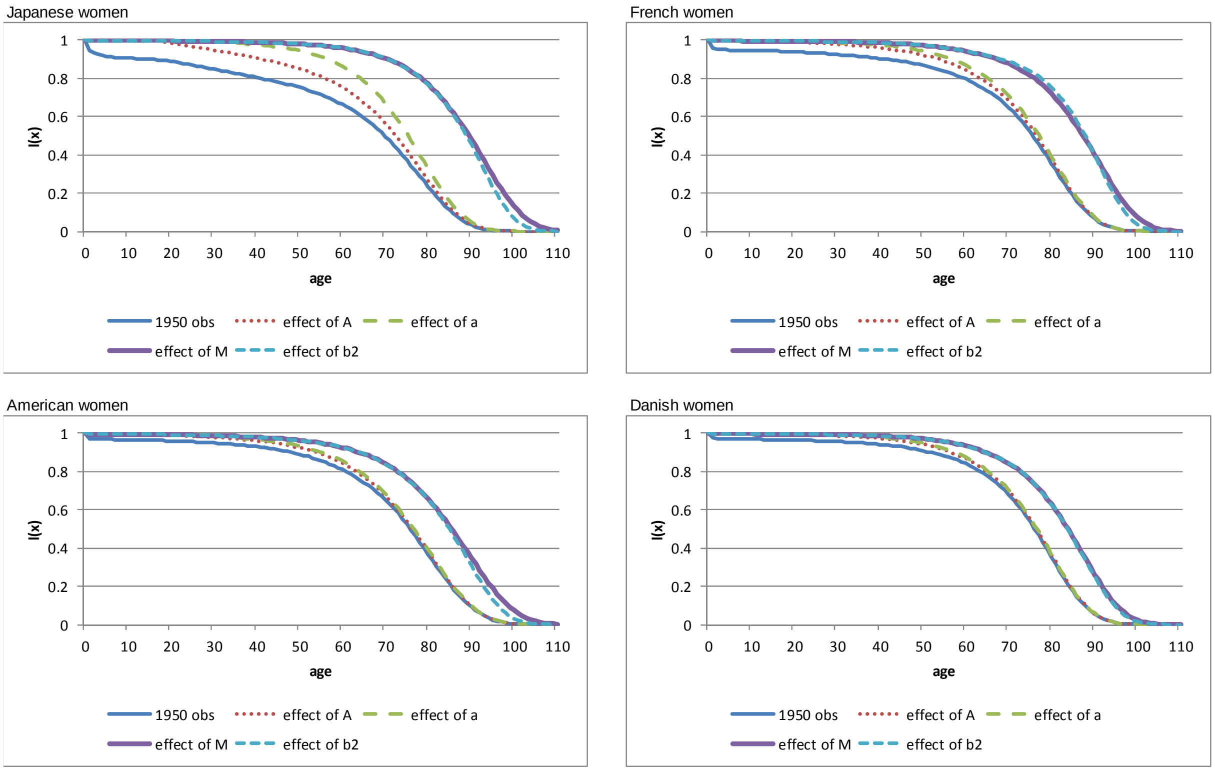


Figure 4. Survival curve, Japanese, French, American and Danish women, effects of changes in the values of the parameters of the NIDI mortality model (equation 3) between 1950 and 2009



Parametric mortality models are aimed to describe the age pattern of mortality in terms of a mathematical function of age. We present a new parametric model to describe mortality for the entire age span: the NIDI mortality model. The model describes mortality in adulthood and advanced age by a mixture of two logistic models. The NIDI mortality model includes four interpretable time-varying parameters, reflecting (i) changes in the shape of the mortality age schedule in young and old age, resulting in compression of mortality, and (ii) a shift of the mortality age schedule to older ages.

Fitting the NIDI model to probabilities of death in 1950 and 2009 for Japan, France, the USA and Denmark, showed a better fit than the well-known Heligman-Pollard model. The four time-varying parameters explain 99 per cent of the change in life expectancy at birth between 1950 and 2009. Shifts in the mortality age schedule explain two thirds of this change.

The NIDI model, thus, is a valid instrument for describing the age pattern of mortality, for disentangling the effects of delay and compression of mortality on the increase in life expectancy, and can serve as a basis for the projection of mortality into the future.

The Netherlands Interdisciplinary Demographic Institute (NIDI) is an institute for the scientific study of population. NIDI research aims to contribute to the description, analysis and explanation of demographic trends in the past, present and future, both on a national and an international scale. The determants and social consequences of these trends are also studied.

NIDI is a research institute of the Royal Academy of Arts and Sciences (KNAW).